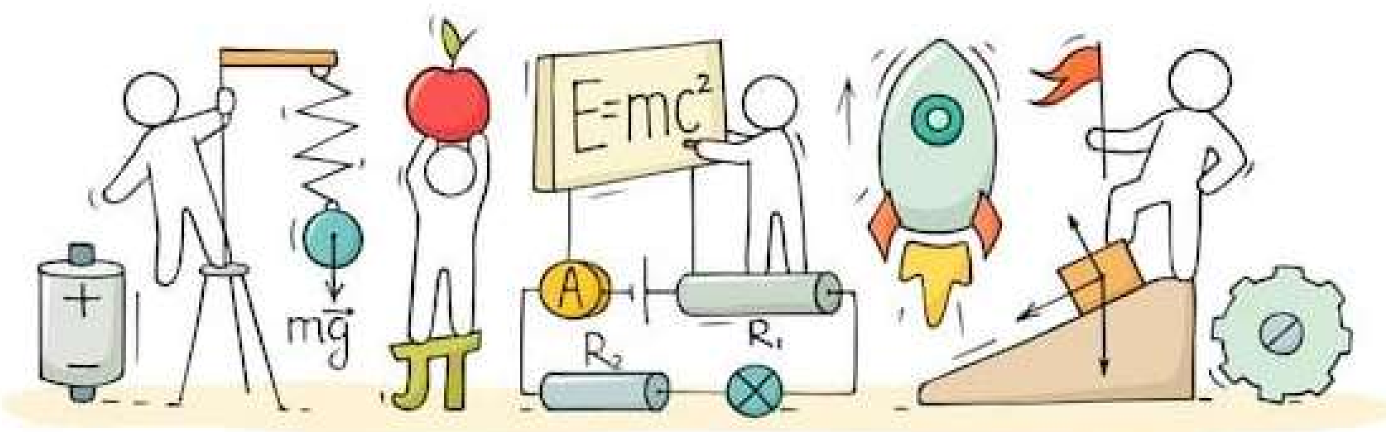
PHYSICS

Chapter 13: Kinetic Theory



Kinetic Theory

Top Formulae

Boyle's law	PV = constant
Charles' law	V/T = constant
Gay-Lussac's law	P/T = constant
Gas equation	$PV = \mu RT$, where μ is the number of moles of the given gas.
Pressure exerted by gas	$P = \frac{1}{3}n m \overline{v^2}$
Mean KE of translation per molecule of a gas	$= \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$
Mean KE of translation per mole of a gas	$= \frac{1}{2}M\overline{v^2} = \frac{3}{2}RT = \frac{3}{2}NkT$
Total KE per mole of gas	$= \frac{n}{2}RT$, where n is the number of degrees of freedom of each molecule.
rms speed	$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}}$
Effect of temperature	$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{}}$
Mean free path	$\vec{l} = \frac{k_B T}{\sqrt{2}\ d\ p} = \frac{1}{\sqrt{2}\pi d^2 n}$ where n is the number of molecules per unit volume of the gas.
Collision frequency	$f = v / \lambda$

Top Concepts

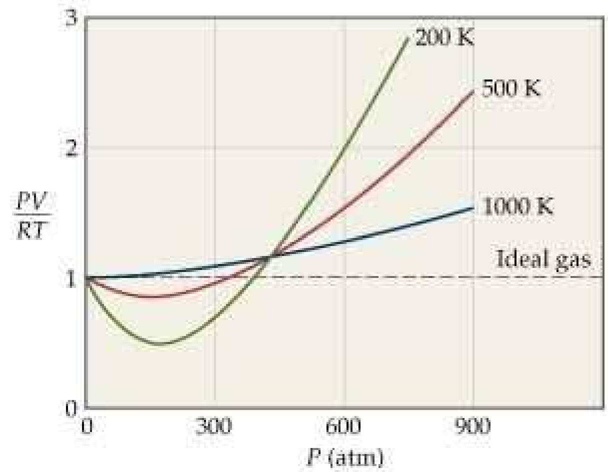
- The kinetic theory of gases relates the macroscopic properties of gases such as pressure, temperature etc. to the microscopic properties of its gas molecules. Examples: Speed, kinetic energy etc.
- An ideal gas is one in which the pressure p, volume V and temperature T are related by $pV = \mu RT$, where R is called the gas constant.
- Real gases satisfy the ideal gas equations only approximately, more so at low pressures and high temperatures.
- The kinetic theory of an ideal gas gives the relation

  $$P = \frac{1}{3} n\, m\overline{v^2}$$

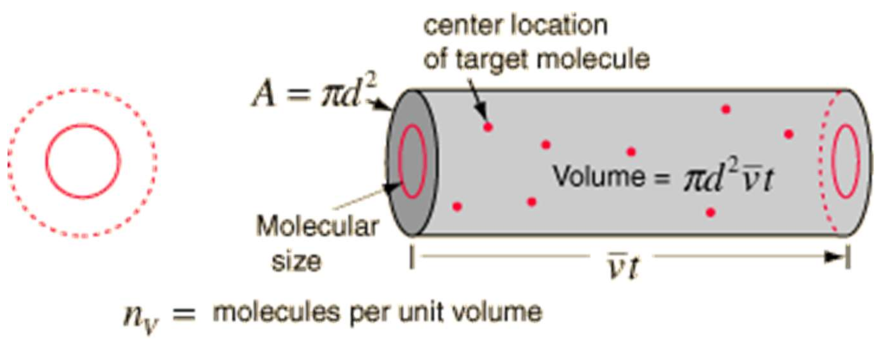
  where n is the number density of molecules, m is the mass of the molecule and $\overline{v^2}$ is the mean of squared speed.
- The temperature of a gas is a measure of the average kinetic energy of molecules independent of the nature of the gas or molecule. In a mixture of gases at a fixed temperature, the heavier molecule has lower average speed.
- The pressure exerted by n moles of an ideal gas in terms of the speed of its molecules is $P = 1/3\ nm\, v_{rms}^2$.
- The average kinetic energy of a molecule is proportional to the absolute temperature of the gas.
- The degrees of freedom of a gas molecule are independent ways in which the molecule can store energy.
- Law of equipartition of energy: Every degree of freedom of a molecule has associated with it, on average, an internal energy of (½) kT per molecule.
- Monoatomic gases only have three translational degrees of freedom.
- Diatomic gases in general have three translational, two rotational and two vibrational degrees of freedom.
- The molar specific heat at constant volume C_v can be written as (f/2)R, where f is the number of degrees of freedom of the ideal gas molecule.

Diagrams

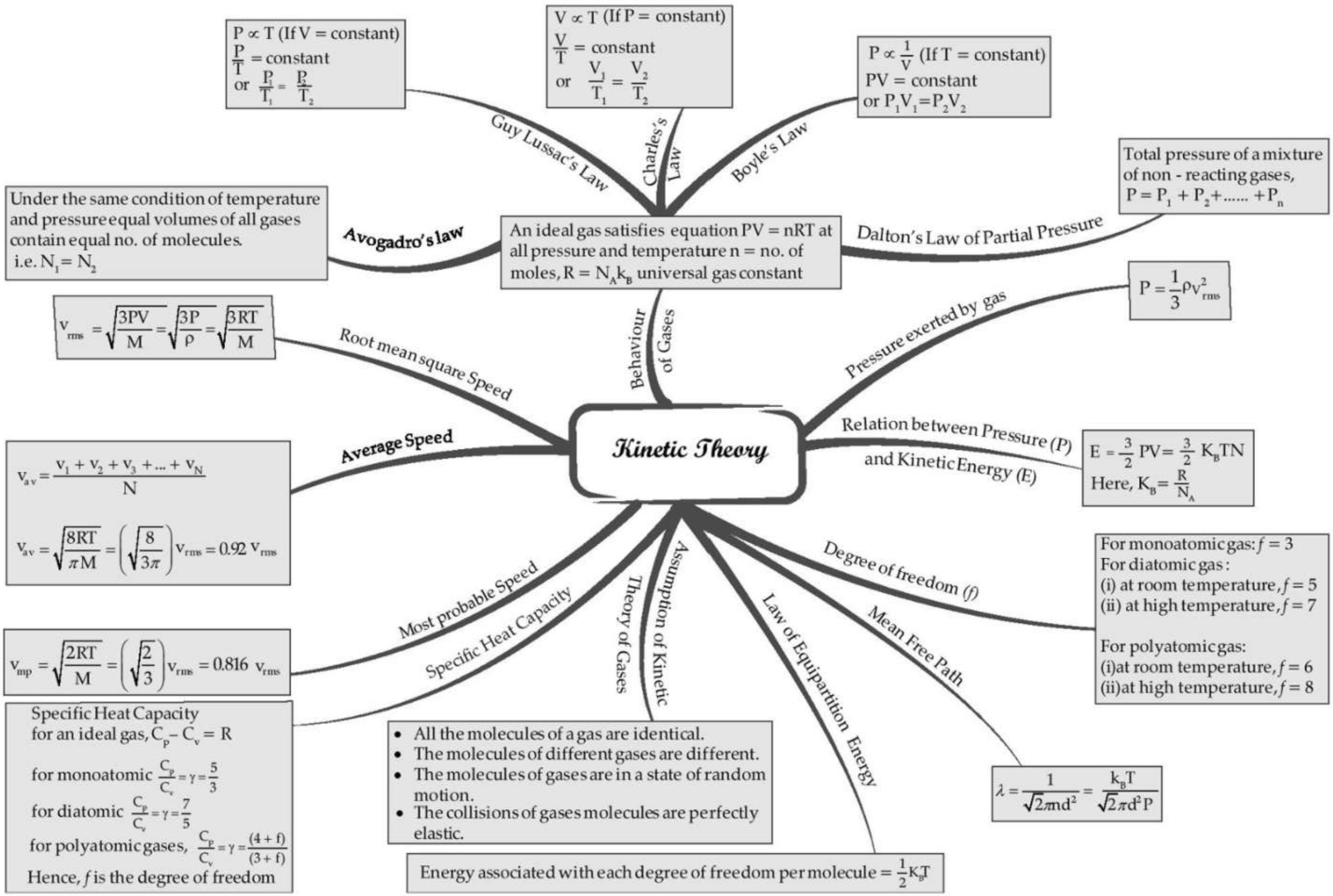
Real gases approach ideal gas behaviour at low pressure and high temperature.



Mean free path



CHAPTER - 13 : KINETIC THEORY



Important Questions

Multiple Choice questions-

Question 1. A room temperature the r.m.s. velocity of the molecules of a certain diatomic gas is found to be 1930 m/sec. the gas is

(a) H^2

(b) F^2

(c) O^2

(d) Cl^2

Question 2. Energy supplied to convert unit mass of substance from solid to liquid state at its melting point is called

(a) Latent heat of fusion

(b) Evaporation

(c) Solidification

(d) Latent heat of fission

Question 3. One any planet, the presence of atmosphere implies [nrms = root mean square velocity of molecules and ne = escape velocity]

(a) nrms << ne

(b) nrms > ne

(c) nrms = ne

(d) nrms = 0

Question 4. Calculate the RMS velocity of molecules of a gas of which the ratio of two specific heats is 1.42 and velocity of sound in the gas is 500 m/s

(a) 727 m/s

(b) 527 m/s

(c) 927 m/s

(d) 750 m/s

Question 5. The r.m.s. speed of the molecules of a gas in a vessel is 200 m/s. if 25% of the gas leaks out of the vessel, at constant temperature, then the r.m.s. speed of the remaining molecules will be

(a) 400 m/s

(b) 150 m/s

(c) 100 m/s

(d) 200 m/s

Question 6. A gas is taken in a sealed container at 300 K. it is heated at constant volume to a temperature 600 K. the mean K.E. of its molecules is

(a) Halved

(b) Doubled

(c) Tripled

(d) Quadrupled

Question 7. Moon has no atmosphere because

(a) It is far away form the surface of the earth

(b) Its surface temperature is 10°C

(c) The r.m.s. velocity of all the gas molecules is more then the escape velocity of the moons surface

(d) The escape velocity of the moons surface is more than the r.m.s velocity of all molecules

Question 8. A unit mass of solid converted to liquid at its melting point. Heat is required for this process is:

(a) Specific heat

(b) Latent heat of vaporization

(c) Latent heat of fusion

(d) External latent heat

Question 9. One mole of ideal gas required 207 J heat to rise the temperature by 10°K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10°K the heat required is (R = 8/3 J/mole °K)

(a) 1987 J

(b) 29 J

(c) 215.3 J

(d) 124 J

Question 10. The r.m.s velocity of the molecules of an ideal gas is C at a temperature of 100K. at what temperature is r.m.s. velocity will be doubted?

(a) 200 K

(b) 400 K

(c) 300 K

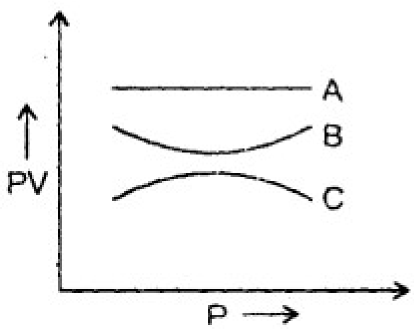
(d) 50 K

Very Short:

1. What does gas constant R signify? What is its value?
2. What is the nature of the curve obtained when:

   (a) Pressure versus reciprocal volume is plotted for an ideal gas at a constant temperature.

   (b) Volume of an ideal gas is plotted against its absolute temperature at constant pressure.
3. The graph shows the variation of the product of PV with the pressure of the constant mass of three gases A, B and C. If all the changes are at a constant temperature, then which of the three gases is an ideal gas? Why?



4. On the basis of Charle’s law, what is the minimum possible temperature?
5. What would be the ratio of initial and final pressures if the masses of all the molecules of a gas are halved and their speeds are doubled?
6. Water solidifies into ice at 273 K. What happens to the K.E. of water molecules?
7. Name three gas laws that can be obtained from the gas equation.
8. What is the average velocity of the molecules of a gas in equilibrium?
9. A vessel is filled with a mixture of two different gases. Will the mean kinetic energies per molecule of both gases be equal? Why?
10. The density of a gas is doubled, keeping all other factors unchanged. What will be the effect on the pressure of the gas?

Short Questions:

1. Why cooling is caused by evaporation?
2. On reducing the volume of the gas at a constant temperature, the pressure of the gas increases. Explain on the basis of the kinetic theory of gases.
3. Why temperature less than absolute zero is not possible?

4. There are n molecules of a gas in a container. If the number of molecules is increased to 2n, what will be:

   (a) the pressure of the gas.

   (b) the total energy of the gas.

   (c) r.m.s. speed of the gas molecules.

5. Equal masses of O_2 and He gases are supplied equal amounts of heat. Which gas will undergo a greater temperature rise and why?

6. Two bodies of specific heats S_1 and S_2 having the same heat capacities are combined to form a single composite body. What is the specific heat of the composite body?

7. Tell the degree of freedom of:

   (a) Monoatomic gas moles.

   (b) Diatomic gas moles.

   (c) Polyatomic gas moles.

8. State law of equipartition of energy.

9. Explain why it is not possible to increase the temperature of gas while keeping its volume and pressure constant?

10. A glass of water is stirred and then allowed to stand until the water stops moving. What has happened to the K.E. of the moving water?

Long Questions:

1. Calculate r.m.s. the velocity of hydrogen at N.T.P. Given the density of hydrogen = 0.09 kg m^4.

2. Calculate the temperature at which r.m.s. the velocity of the gas molecule is double its value at 27°C, the pressure of the gas remaining the same.

3. Calculate the K.E./mole of a gas at N.T.P. Density of gas at N.T.P. = 0.178 g dm^{-3} and molecular weight = 4.

4. Calculate the diameter of a molecule if $n = 2.79 \times 10^{25}$ molecules per m3 and mean free path = 2.2×10^{-8} m.

5. Calculate the number of molecules in 1 cm^3 of a perfect gas at 27°C and at a pressure of 10 mm of Hg. Mean K.E. of a molecule at 27°C = 4 × 1025 J. ρ_{Hg} = 13.6 × 103 kg m^{-3}.

Assertion Reason Questions:

1. **Directions:** Choose the correct option from the following:

   (a) Both A and R are true, and R is the correct explanation of A
   (b) Both A and R are true, but R is NOT the correct explanation of A

(c) A is true but R is false
(d) A is false and R is also false

Assertion (A): The number of degrees of freedom of a linear triatomic molecules is 7.
Reason (R): The number of degrees of freedom depends on number of particles in the system.

2. **Directions:** Choose the correct option from the following:

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

Assertion (A): Absolute zero is not the temperature corresponding to zero energy.
Reason (R): The temperature at which no molecular motion ceases is called absolute zero temperature.

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (a) H^2
2. Answer: (a) Latent heat of fusion
3. Answer (a) nrms << ne
4. Answer: (a) 727 m/s
5. Answer: (d) 200 m/s
6. Answer: (b) Doubled
7. Answer: (c) The r.m.s. velocity of all the gas molecules is more then the escape velocity of the moons surface
8. Answer: (c) Latent heat of fusion
9. Answer: (d) 124 J
10. Answer: (b) 400 K

Very Short Answers:

1. Answer: The universal gas constant (R) signifies the work done by (or on) a gas per mole per kelvin. Its value is 8.31 J mol^{-1} K
2. Answer: (a)It is a straight line.

   (b) It is a straight line.
3. Answer: A is an ideal gas because PV is constant at constant temperature for an ideal gas.
4. Answer: – 273.15°C.

5. Answer: 1: 2 ($\because P = \frac{13mn}{V} C^2$)
6. Answer: It is partly converted into the binding energy of ice.
7. Answer:
   1. Boyle's law
   2. Charle's law
   3. Gay Lussac's law.
8. Answer: Zero.
9. Yes. This is because the mean K.E. per molecule i.e. $\frac{3}{2}kT$ depends only upon the temperature.
10. It will be doubled. ($\because P \propto \rho$ if other factors are constant).

Short Questions Answers:

1. Answer: Evaporation occurs on account of faster molecules escaping from the surface of the liquid. The liquid is therefore left with molecules having lower speeds. The decrease in the average speed of molecules results in lowering the temperature and hence cooling is caused.
2. Answer: On reducing the volume, the space for the given number of molecules of the gas decreases i.e. no. of molecules per unit volume increases. As a result of which more molecules collide with the walls of the vessel per second and hence a larger momentum is transferred to the walls per second. Due to which the pressure of gas increases.
3. Answer: According to the kinetic interpretation of temperature, absolute temperature means the kinetic energy of molecules.

   As heat is taken out, the temperature falls and hence velocity decreases. At absolute zero, the velocity of the molecules becomes zero i.e. kinetic energy becomes zero. So no more decrease in K.E. is possible, hence temperature cannot fall further.
4. Answer: (a) We know that

   $P = \frac{1}{3}mnC^2$.

   where n = no. of molecules per unit volume.

   Thus when no. of molecules is increased from n to 2n, no. of molecules per unit volume (n) will increase from n 2n

   $\frac{n}{V}$ to $\frac{2n}{V}$, hence pressure will become double.

   (b) The K.E. of a gas molecule is,

   $\frac{1}{2}mC^2 = \frac{3}{2}kT$

   If the no. of molecules is increased from n to 2n. There is no effect on the average K.E. of a gas molecule, but the total energy is doubled.

r.m.s speed of gas is $C_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3P}{mn}}$

When n is increased from n to 2n. both n and P become double and the ratio $\frac{P}{n}$ remains unchanged. So there will be no effect of increasing the number of molecule from n to 2n on r.m.s. speed of gas molecule.

5. Answer: Helium is monoatomic while O_2 is diatomic. In the case of helium, the supplied heat has to increase only the translational K.E. of the gas molecules.

   On the other hand, in the case of oxygen, the supplied heat has to increase the translations, vibrational and rotational K.E. of gas molecules. Thus helium would undergo a greater temperature rise.

6. Answer: Let m_1 and m_2 be the masses of two bodies having heat capacities S1 and S1 respectively.

   $\therefore (m_1 + m_2)S = m_1S_1 + m_2S_2 = m_1S_1 + m_1S_1 = 2m_1S_1$

   $S = \frac{2m_1S_1}{m_1+m_2}$.

   Also, $m_2S_2 = m_1S_1$

   (∵ Heat capacities of two bodies are same.)

   Or

   $m_2 = \frac{m_1 S_1}{S_2}$

   $\therefore S = \frac{2 m_1 S_1}{m_1 + \frac{m_1 S_1}{S_2}} = \frac{2 S_1 S_2}{S_1+S_2}$

7. Answer: (a) A monoatomic gas possesses 3 translational degrees of freedom for each molecule.

   (b) A diatomic gas molecule has 5 degrees of freedom including 3 translational and 2 rotational degrees of freedom.

   (c) The polyatomic gas molecule has 6 degrees of freedom (3 translational and 3 rotational).

8. Answer: It states that in equilibrium, the total energy of the system is divided equally in all possible energy modes with each mode i.e. degree of freedom having an average energy equal to $\frac{1}{2}$ K_BT.

9. Answer: It is not possible to increase the temperature of a gas keeping volume and pressure constant can be explained as follows:

   According to the Kinetic Theory of gases,

$$P = \frac{1}{3}\rho C^2 = \frac{1}{3}\frac{M}{V}C^2$$

$$= \frac{1}{3}\frac{M}{V}kT$$

(∵ $C^2 = kT$, when k is a constant)

$T \propto PV$

Now as T is directly proportional to the product of P and V. If P and V are constant, then T is also constant.

10. Answer: The K.E. of moving water is dissipated into internal energy. The temperature of water thus increases.

Long Questions Answers:

1. Answer:

Here,

$$\rho = 0.09 \text{ kg m}^{-3}$$

$$P = 76 \text{ cm of Hg}$$

$$= 76 \times 13.6 \times 980 \text{ dyne cm}^{-2}$$

$$= 1.01 \times 10^6 \text{ dyne cm}^{-2}$$

$$= 1.01 \times 10^5 \text{ Nm}^{-2}$$

$$C = ?$$

Using the relation, $P = \frac{1}{3}\rho C^2$, we get

$$C = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{0.09}}$$

$$= \sqrt{3.37 \times 10^6}$$

$$C = 1.836 \times 10^3 \text{ ms}^{-1}$$

$$= 1836 \text{ ms}^{-1}.$$

2. Answer: Let t be the required temperature = ? and Ct, C27 be the r.m.s. velocities of the gas molecules at t°C and 27°C respectively.

$\frac{C_t}{C_{27}}$ = 2 (given)

Also let M = molecular weight of the gas

Now T = t + 273

and T27 = 27 + 273 = 300 K

∴ Using the relation

$$C = \sqrt{\frac{3RT}{M}}$$, we get

$$C_t = \sqrt{\frac{3RT}{M}}$$

and $$C_{27} = \sqrt{\frac{3RT_{27}}{M}}$$

$$\therefore \quad \frac{C_t}{C_{27}} = \sqrt{\frac{T}{T_{27}}} = \sqrt{\frac{t+273}{300}}$$

or $$2 = \sqrt{\frac{t+273}{300}}$$

or $$4 = \frac{t+273}{300}$$

or $$t = 1200 - 273 = 927°C.$$

3. Answer: Here, $\rho = 0.178$ g dm^{-3}

$= 0.178 \times 10^{-3}$ g cm^{-3} ($\because$ 1 $dm^3 = 10^{-3}$ cm^3)

$= 178 \times 10^{-6}$ g cm^{-3}

Volume of 1 mole of gas i.e. 4 g of gas $= \frac{\text{Mass}}{\text{Density}}$

or $$V = \frac{M}{\rho} = \frac{4}{178\times10^{-6}} \text{ cm}^3$$

T = 273 K at NTP

$$\therefore \quad R = \frac{PV}{T} = \frac{76\times13.6\times980\times4}{178\times10^{-6}\times273}$$

K.E./mole = ?

We know that $$\text{K.E./mole} = \frac{3}{2}RT$$

$$= \frac{3}{2} \times \frac{76\times13.6\times980\times4}{178\times10^{-6}\times273} \times 273$$

$$= 3.42 \times 10^{10} \text{ erg}$$

$$= \frac{3.42\times10^{10}}{10^7} \text{ J} = 3.42 \times 10^3 \text{ J.}$$

4. Answer: Here, $n = 2.79 \times 10^{25}$ molecules m^{-3}

$\lambda = 2.2 \times 10^{-8}$ m

d = ?

Using the relation.

$$\lambda = \frac{1}{\sqrt{2}}\frac{1}{\pi n d^2}\text{, we get}$$

$$d^2 = \frac{1}{\sqrt{2}}\frac{1}{\pi n \lambda}$$

$$= \frac{1}{1.414\times3.142\times2.79\times10^{25}\times2.2\times10^{8}}$$

$$= 0.03666 \times 10^{-7}\ m^2$$

$$= 0.367 \times 10^{-18}\ m^2$$

$$\therefore \quad d = \sqrt{0.367\times10^{-18}\,m^2}$$

$$= 0.606 \times 10^{-9}\ m = 606\ nm.$$

5. Answer: Here, K..E. per molecule at 27°C = 4×10^{-11} J

Let μ = number of molecules in 1 cm^3 or 10-6 m^3

∴ Mean K.E. per cm3 = μ × 4 × 1011 J ….(i)

Now K.E. per gram molecule = $\frac{3}{2}$ RT

for a perfect gas, PV = RT

∴ K.E, per gram molecule = $\frac{3}{2}$ PV

or

K.E. per cm3 of gas = $\frac{3}{2}$ PV

P = 10 mm of Hg = 10^{-2} m of Hg

= $10^{-2} \times 13.6 \times 10^3 \times 9.8$

= $136 \times 9.8\ Nm^{-2}$ V

= 1 cm^3

= $10^{-6}\ m^3$

∴ K.E per cm^3 of gas = $\frac{3}{2} \times 136 \times 9.8 \times 106$

= 1.969×10^{-3} J ….(ii)

∴ from (i) and (ii) we get

$\mu \times 4 \times 10^{-11} = 1.969 \times 10^{-3}$

or

$$\mu = \frac{1.969 \times 10^{-3}}{4 \times 10^{11}}$$

= 4.92 × 107 molecules.

Assertion Reason Answer:

1. (b) Both A and R are true, but R is NOT the correct explanation of A
2. (a) Both A and R are true, and R is the correct explanation of A

Case Study Questions-

1. Boyle's law is a gas law which states that the pressure exerted by a gas (of a given mass, kept at a constant temperature) is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept constant. For a gas, the relationship between volume and pressure (at constant mass and temperature) can be expressed mathematically as follows. P ∝ (1/V) Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant, k. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure. The law also states that the Kelvin temperature and the volume will be in direct proportion when the pressure exerted on a sample of a dry gas is held constant. Charles law and Boyle's law applied to low density gas only. The total pressure of a mixture of ideal gases is the sum of partial pressures. This is Dalton's law of partial pressures.

   i. Boyle's law is obeyed by high as well as low density gases. True or False?
      a. True
      b. False

   ii. Charles law is states that volume of an ideal gas is directly proportional to temperature at constant
      a. Temperature
      b. Pressure
      c. Volume
      d. None of these

   iii. State Daltons law of partial pressures

   iv. State Boyle's law

   v. State Charles law

2. Pressure of an Ideal Gas: according to kinetic theory of gases pressure is given by P = 1/3 nmv2 Where, n is number of molecules per unit volume, m is mass and v2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial. The

average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature, a macroscopic measurable parameter of a gas (a thermodynamic variable as it is called) to a molecular quantity, namely the average kinetic energy of a molecule. The two domains are connected by the Boltzmann constant and given by E = kbT. Where kb is Boltzmann constant having value of 1.38*10-23 joule per Kelvin. We have seen that in thermal equilibrium at absolute temperature T, for each translational mode of motion, the average energy is ½ Kb T. The most elegant principle of classical statistical mechanics (first proved by Maxwell) states that this is so for each mode of energy: translational, rotational and vibrational. That is, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to ½ kB T. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes ½ kB T to the energy, while each vibrational frequency contributes 2 × ½ kB T = kB T, since a vibrational mode has both kinetic and potential energy modes.

i. Boltzmann constant has value of
   a. $1.38*10^{-23}$ joule per Kelvin.
   b. $1.38*10^{-28}$ joule per Kelvin.
   c. $1.38*10^{-30}$ joule per Kelvin.
   d. None of these
ii. SI unit of Boltzmann constant is given by
   a. Joules per meter
   b. Joules per Kelvin
   c. Joules per Newton
   d. None of these
iii. According to kinetic theory give formula for pressure of idea gas.
iv. According to kinetic theory what is average kinetic energy of molecules of ideal gas?
v. What is law of equipartition of energy?

Case Study Answer-

1. **Answer**

i. (a) True
ii. (b) Pressure
iii. The total pressure of a mixture of ideal gases is the sum of partial pressures exerted by all the molecules of gas. This is Dalton’s law of partial pressures.

iv. Boyle's law is a gas law which states that at constant temperature the pressure exerted by a gas is inversely proportional to the volume occupied by it. In other words, the pressure and volume of a gas are inversely proportional to each other as long as the temperature and the quantity of gas are kept constant. For a gas, the $P \propto (1/V)$ Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant k.

v. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure.

2. Answer

i. (a) $1.38*10^{-23}$ joule per Kelvin.

ii. (b) Joules per Kelvin

iii. According to kinetic theory of gases pressure is given by $P = 1/3\ nmv^2$ Where, n is number of molecules per unit volume, m is mass and v^2 is mean squared speed. Though we choose the container to be a cube, the shape of the vessel really is immaterial.

iv. The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas and given by $E = 3/2\ k_bT$.
Where k_b is Boltzmann constant having value of $1.38*10^{-23}$ joule per Kelvin.

v. We know that for each translational mode of motion, the average energy is ½ Kb T. classical statistical mechanics states that in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to ½ k_BT. This is known as the law of equipartition of energy. Accordingly, each translational and rotational degree of freedom of a molecule contributes ½ k_BT to the energy, while each vibrational frequency contributes $2 \times ½\ k_B T = k_BT$, since a vibrational mode has both kinetic and potential energy modes.